Irrationality of Lambert series associated with periodic sequence

Yohei Tachiya

This talk is based on a joint work with Florian Luca.

Let q be an integer with $|q| > 1$ and $\{a_n\}_{n\geq 1}$ be an eventually periodic sequence of rational numbers, not identically zero from some point on. Then the number $\sum_{n=1}^{\infty} a_n/(q^n-1)$ is irrational. In particular, if the periodic sequences $\{a_n^{(i)}\}_{n\geq 1}$ $(i=1,\dots,m)$ of rational numbers are linearly independent over $\mathbb{Q}$, then so are the following $m+1$ numbers:

$$1, \quad \sum_{n=1}^{\infty} \frac{a_n^{(i)}}{q^n-1}, \quad i=1,\dots,m.$$

This generalizes a result of Erdős who treated the case $m=1$ and $a_n^{(1)}=1$ $(n\geq 1)$.